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06MAT41

Fourth Semester B.E. Degree Examination, May/June 2010
Engineering Mathematics - IV

Time: 3 hrs. Max. Marks:100

Note: *1.Answer any FIVE full questions, selecting at least TWO questions from each part.*
2.Use of statistical tables is permitted.

PART – A

1 a. Find the y(0.1) correct to 6 decimal places by Taylor series method when dy/dx = xy + 1, y(0) = 1.0. (Consider upto 4th degree term). **(06 Marks)**

b. Using Runge-Kutta method of order 4, compute y(0.2) for the equation, $y' = y - \frac{2x}{y}$, y(0) = 1.0 (Take h = 0.2). **(07 Marks)**

c. Given that $y' = x^2(1+y)$ and y(1) = 1.0, y(1.1) = 1.233, y(1.2) = 1.548 and y(1.3) = 1.979, compute y(1.4) by Adams-Bashforth method. Apply correct formula twice. **(07 Marks)**

2 a. Show that Z^n is analytic. Hence find its derivative. **(06 Marks)**

b. Find a bilinear transformation which maps the points 0, 1, i in the Z-plane onto 1 + i, –i, 2 – i in the W plane. **(07 Marks)**

c. Find the analytic function u + iv, where u is given to be $u = e^x[(x^2 - y^2)\cos y - 2xy \sin y]$. **(07 Marks)**

3 a. Derive Couchy's integral formula in the form

$$f(a) = \frac{1}{2\pi i}\int_c \frac{f(z)dz}{z-a}$$ **(06 Marks)**

b. Expand $f(z) = \frac{7z^2 + 9z - 18}{z^3 - 9z}$ in the Laurent series that is valid for

i) $|z| > 3$ ii) $0 < |z-3| < 3$. **(07 Marks)**

c. Evaluate $\int_c \tan z\, dz$, where c is $|z| = 2.5$ **(07 Marks)**

4 a. Find the series solution of $\frac{d^2y}{dx^2} + xy = 0$. **(06 Marks)**

b. Express $x^4 + 3x^3 - x^2 + 5x - 2$ in terms of Legendre's polynomials. **(07 Marks)**

c. Reduce the differential equation $x\frac{d^2y}{dx^2} + \alpha\frac{dy}{dx} + k^2xy = 0$ to Bessel's equation. Obtain the solution. **(07 Marks)**

PART – B

5 a. Fit a curve of the form $y = ab^x$ for the data given below: **(06 Marks)**

x :	2	4	6	8	10	12
y :	1.8	1.5	1.4	1.1	1.1	0.9

b. Find the coefficient of correlation for the following data: **(07 Marks)**

x :	55	56	58	59	60	60	62
y :	35	38	39	38	44	43	45

c. In a certain college 25% of boys and 10% of girls are studying mathematics. The girls constitute 60% of the student body.
   i) What is the probability that mathematics is being studied?
   ii) If a student is selected a random and is found to be studying mathematics, find the probability that the student is a girl. **(07 Marks)**

6 a. Suppose a random variable X takes the values –3, –1, 2 and 5 with respective probabilities $\frac{2k-3}{10}, \frac{k-2}{10}, \frac{k-1}{10}, \frac{k+1}{10}$. Find the value of k and i) find $P[-3 < X < 4]$ and ii) $P[X \leq 2]$. **(06 Marks)**

b. Suppose that the student IQ scores form a normal distribution with mean 100 and standard deviation 20. Find the percentage of students whose i) score is less than 80 ii) score falls between 90 and 140, iii) Score more than 120. **(07 Marks)**

c. Obtain mean and variance of binomial distribution function. **(07 Marks)**

7 a. A sample of 1000 days is taken from meteorological records of a certain district and 120 of them are found to be foggy. What are the probable 99% confidence limits to the proportion of foggy days in the district? **(06 Marks)**

b. The following table gives the number of bus accidents that occurred during the various days of the week. Find whether the accidents are uniformly distributed over the week, using χ^2 test. **(07 Marks)**

Days	Sun	Mon	Tue	Wed	Thu	Fri	Sat	Total
No. of accidents	14	16	8	12	11	9	14	84

c. The life X of certain computer is approximately normally distributed with mean 800 hours and standard deviation 40 hours. If a random sample of 30 computers has an average life of 788 hours, test the hypothesis that $\mu = 800$ hours against the alternate hypothesis $\mu \neq 800$ hours at i) 0.5% and 1% level of significance. **(07 Marks)**

8 a. A fair coin is tossed 4 times. Let X denote the number of heads occurring and let Y denote the longest string of heads occurring. Find the joint distribution function of X and Y. **(06 Marks)**

b. A man's gambling luck follows a pattern. If he wins a game the probability of winning the next game is 0.6. However, if he loses a game, the probability of losing the next game is 0.7. There is an even chance that he wins the first game.
i) Find the transition matrix of the Markov process. ii) Find the probability that he wins the third game. iii) Find out how often, in the long run, he wins. **(07 Marks)**

c. Explain: i) Transient state ii) Absorbing state and iii) Recurrent state by means of an example each. **(07 Marks)**

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06ES42

Fourth Semester B.E. Degree Examination, May/June 2010
Micro–Controllers

Time: 3 hrs. Max. Marks:100

Note: *1. Answer any FIVE full questions selecting at least TWO questions from each part.*
2. Standard notations are used.

PART – A

1 a. Give the basic block diagrams, of a microprocessor and a microcontroller and justify that a microcontroller is an onchip computer. **(08 Marks)**

b. What is Harvard architecture? Show that 8051 uses Harvard architecture. **(06 Marks)**

c. Briefly discuss the features of 8051 microcontroller. **(06 Marks**

2 a. Explain the following in brief, with respect to 8051 :
i) The pin that connects the external memory
ii) The port that has open-drain output
iii) Asynchronous input pins of microcontrollers
iv) The register that sequences the program execution
v) Program status word. **(10 Marks)**

b. Write the circuit diagram for Port1. Explain the input, output operations in 8051 using Port1. **(10 Marks)**

3 a. Give the mode word, (TMOD) and the control word, (TCON) values to perform the following operations :
i) Timer 0 in auto reload mode
ii) Timer 1 in mode1. **(06 Marks)**

b. Explain the serial data interrupts TI and RI in 8051. **(06 Marks)**

c. Name the addressing modes of the following instructions :
i) MOVC A, @ A + DPTR
ii) MULAB
iii) MOV B, #OFFh
iv) SUBB A, 45h. **(04 Marks)**

d. Explain any two data transfer instructions and any one arithmetic instruction in 8051. **(04 Marks)**

4 a. Name the instructions which perform bit level logical operations in 8051. Give an example to show bit level logic operation. **(06 Marks)**

b. Write an assembly program in 8051 to add two 16 bit numbers stored in external memory. After addition store the results in internal data memory. **(06 Marks)**

c. Write the result statement after execution of each instruction :
MOV 81 h, #30 h
MOV RO, #OAC h
PUSH 00
PUSH 00
POP 01
POP 80 h
MOV A, # OFF h
XRL A, 80 h
POP 82 h
POP 83 h
MOVX @ DPTR, A. **(08 Marks)**

PART – B

5 a. Write an assembly program in 8051 to convert a given two digit hexadecimal number to its equivalent decimal number and send the result on to port 2 byte by byte. **(06 Marks)**

b. How is a 'call' subroutine different from an interrupt service routine? Give an example to show call subroutine' operation in 8051. **(06 Marks)**

c. What are the final numbers in A, B and OV flag after the execution?

```
MOV A, # 7B h
MOV OFO, #02 h
MUL AB
MOV B, # OFE h
MUL AB.
```

(04 Marks)

d. Give the magnitude of different data types used in embedded 'C'. **(04 Marks)**

6 a. Write the block diagram to show mode 2 operation of timer 1, as a counter, also write the programming steps to perform the same. **(06 Marks)**

b. Find the delay generated by timer O in the following code. Calculate the delay generated excluding the instruction overhead. What count has to be loaded in TLO and THO if delay has to be increased to 25 msec?

```
        CLR P2.3
HERE : MOV TMOD, #01
       MOV TLO, # 3E h
       MOV THO, # 0 B8 h
       SETB P2.3
       SETB TRO
AGAIN : JNB TFO, AGAIN
        CLR TFO
        CLR TRO
        CLR P2.3.
```

(08 Marks)

c. What is asynchronous serial communication? Explain the different modes of serial communication in 8051. **(06 Marks)**

7 a. Write 8051 'C' program to receive bytes of data serially and put them in P1. Set the baud rate as 4800, 8 bit data and one stop bit. **(06 Marks)**

b. Explain the different interrupt vector addresses in 8051. **(04 Marks)**

c. Write a 8051 'C' program that continuously gets a single bit of data from P1.7 and sends it to P1.0, which creates a square wave of 200 μs period on pin P2.5. XTAL frequency = 11.0592 MHz. **(10 Marks)**

8 a. Interface LCD to 8051 and write a 8051 assembly /8051 'C' program to send 'M', 'A' 'S', 'T', 'E', 'R' to LCD display. **(10 Marks)**

b. Show the interfacing of a stepper motor to 8051 and write 8051 assembly/8051 'C' program to rotate stepper motor 2 rotations clockwise and one rotation anticlockwise with appropriate delay. **(10 Marks)**

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06ES43

Fourth Semester B.E. Degree Examination, May/June 2010
Control Systems

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions, selecting at least TWO questions from each part.***

PART – A

1 a. Distinguish between open loop and closed loop control systems. Give two examples for each. **(06 Marks)**

b. Write the differential equations for the mechanical rotational system shown in Fig. Q1(b) below. Obtain the torque-current analogy of the system. List all the analogous quantities. **(07 Marks)**

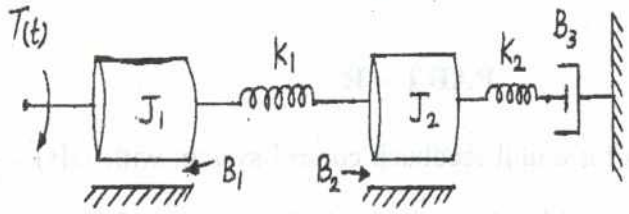

Fig. Q1(b)

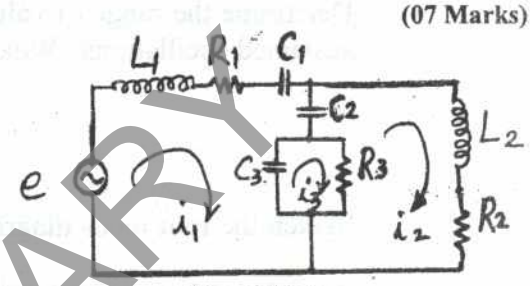

Fig. Q1(c)

c. The force-voltage analogy of a mechanical translational system is given in Fig.Q. 1(c). Obtain its analogous mechanical system. **(07 Marks)**

2 a. Obtain the overall T.F C/R, by the block diagram reduction techniques. Refer Fig. Q2(a). **(10 Marks)**

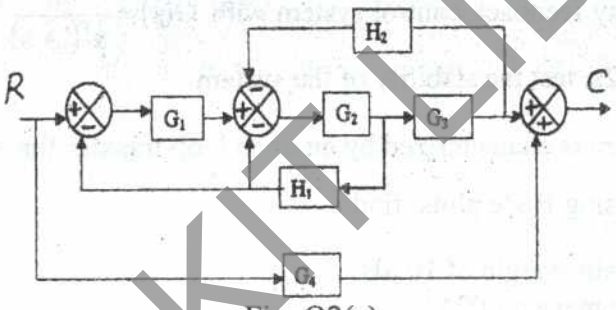

Fig. Q2(a)

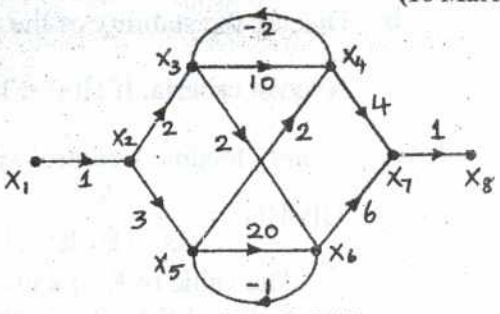

Fig.Q.2 (b)

b. Determine the gain X_8/X_1, of the system described by the signal flow graph given in Fig.Q.2 (b). **(10 Marks)**

3 a. Derive an expression for the unit step response of an underdamped second order system, with usual notations. Sketch this response and mark clearly the rise time t_r, peak time t_p, settling time t_s and peak overshoot M_p. **(08 Marks)**

b. A unity feedback control system is characterized by an open loop T.F, $G(s) = \frac{K}{s(s+\alpha)}$.

Where K and α are positive constants.

i) By what factor the amplifier gain K should be reduced so that the peak overshoot of the unit step response reduces from 75% to 25%?

ii) Find the values of K and α, so that, damping ratio is 0.6 and frequency of damped oscillations is 8 rad / sec. Also find the peak value of the response, when the system is excited by a step of 2V.

iii) If the above open loop T.F. G(s) is multiplied with a factor $(s+\beta)$, and the closed loop poles are located at $-1 \pm j1$, find the values of K, α and β so that the steady state value of error for a ramp input equals 1/10. **(12 Marks)**

4 a. Determine the values of k and a, so that the system shown in Fig.Q.4(a), oscillates at a frequency of 2 rad / sec. **(10 Marks)**

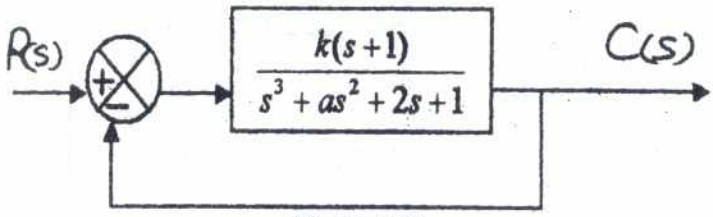

Fig.Q.4(a)

b. The open loop T.F. of a unity feedback control system is given by

$$G(s)=\frac{K}{(s+2)(s+4)(S^2+6s+25)}.$$

Determine the range of values of K for system stability. What is the value of K which gives sustained oscillations? What is the oscillation frequency? **(10 Marks)**

PART – B

5 Sketch the root locus diagram for a unit feedback control system with $G(s)=\frac{K}{s(s^2+8s+17)}$ using the rules of construction and by determining the break away /break in points and the angle of departure. Find the value of K for which the system just oscillates. From the root locus, determine the value of K for a damping ratio of 0.5. **(20 Marks)**

6 a. State and explain Nyquist stability criteria. **(06 Marks)**

b. Discuss the stability of the unity feedback control system with $G(s)=\frac{1}{s^2(1+s)}$ by using Nyquist criteria. If H(s) = 1 + 2s, test the stability of the system. **(14 Marks)**

7 a. A unity feedback control system is characterized by an open loop transfer function $G(s)H(s)=\frac{K}{s(s+1)(0.1s+1)}$. Using Bode plots, find :

i) The value of K to a give gain margin of 10 dB

ii) Value of K to give a phase margin of 24°. **(12 Marks)**

b. Find the T.F of the system whose Bode diagram is given in Fig. Q7(b) **(08 Marks)**

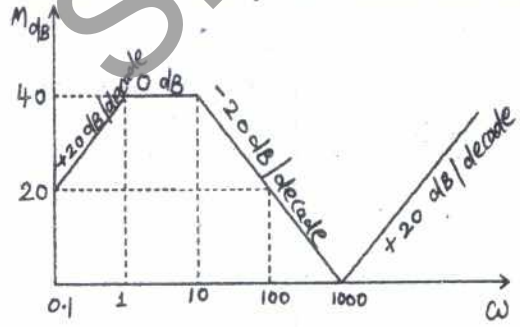

Fig Q7(b)

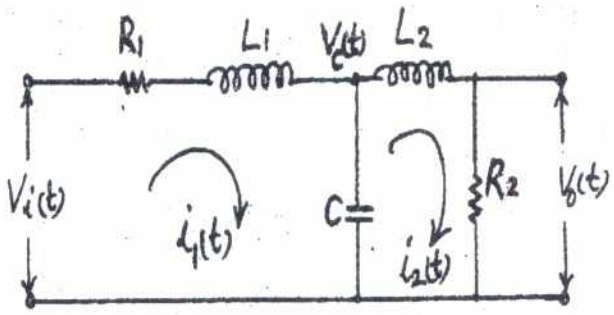

Fig. Q8(b)

8 a. Define state variable and state transition matrix. List the properties of the state transition matrix. **(08 Marks)**

b. Obtain the state model for the electrical system given in Fig. Q 8(b), choosing the state variables as $i_1(t)$, $i_2(t)$ and $v_c(t)$. **(12 Marks)**

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06EC44

Fourth Semester B.E. Degree Examination, May/June 2010
Signals and Systems

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions, selecting at least TWO questions from each part.***

PART – A

1 a. Give the classification of signals. **(08 Marks)**

b. Determine and sketch the even and odd part of the signals shown in Fig.Q1(b). **(12 Marks)**

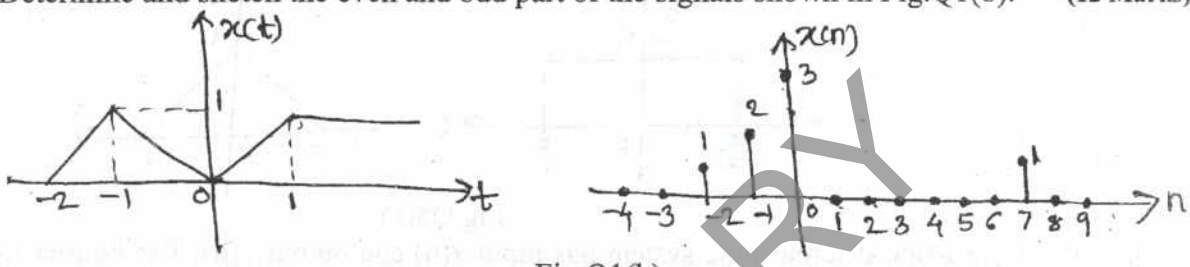

Fig.Q1(b)

2 a. Derive the expression for convolution integral. **(05 Marks)**

b. Verify which of the following systems are linear, causal and invertible:

i) $y(t) = ax(t) + b$ ii) $y(t) = x^2(t)$ iii) $y(n) = \sqrt{x(n)}$ iv) $y(n) = x(4n + 1)$. **(10 Marks)**

c. For a discrete LTI (DLTI) system to be BIBO stable,

Show that $S \triangleq \sum_{K=-\infty}^{\infty} |h(k)| < \infty$ **(05 Marks)**

3 a. By direct evaluation of convolution sum, determine the step response of a discrete system whose unit impulse response $h(n) = (\frac{1}{2})^{-n} u(-n)$. Sketch the response and hence verify whether the system is stable and causal. **(08 Marks)**

b. Obtain $x(t) * y(t)$ for the signals

$x(t) = u(t) - u(t-2)$

$y(t) = t\,[u(t) - u(t-1)]$

Sketch the convolved signal $x(t) * y(t)$. **(08 Marks)**

c. Draw block diagram representations for causal LTI systems whose input output relation is

i) $y(n) = \frac{1}{3}y(n-1) + \frac{1}{2}x(n)$ ii) $y(n) = \frac{1}{3}y(n-1) + x(n-1)$

iii) $y(t) = -\frac{1}{2}\frac{dy(t)}{dt} + 4x(t)$ iv) $\frac{dy(t)}{dt} + 3y(t) = x(t)$ **(04 Marks)**

4 a. Give the significance of time and frequency domain representation of signals. Give examples. **(04 Marks)**

b. Find the CT exponential FS of the signal shown in Fig.Q4(b). **(10 Marks)**

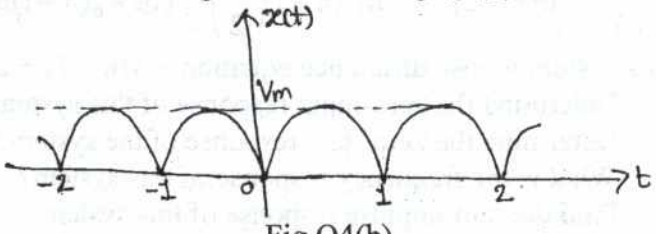

Fig.Q4(b)

c. State and prove the periodic time shift and periodic time convolution properties of DTFS (Discrete time Fourier series). **(06 Marks)**

PART – B

5 a. Obtain the DTFT of the following DT aperiodic sequences:
i) $x(n) = \delta(n) - 3\delta(n-3) + 2\delta(n-4)$ ii) $x(n) = (1/2)^n u(n) - (1/3)^n u(-n-3)$
iii) $x(n) = nu(n) - u(n-1)$ iv) $x(n) = \cos w_o n\, u(n)$ **(04 Marks)**

b. State and prove the Parseval's relation for DTFT. What is the significance of this relation? **(06 Marks)**

c. Using the time differentiation property of CTFT, find the spectrum of the following signals as shown in Fig.Q5(c). Plot the spectrum. **(10 Marks)**

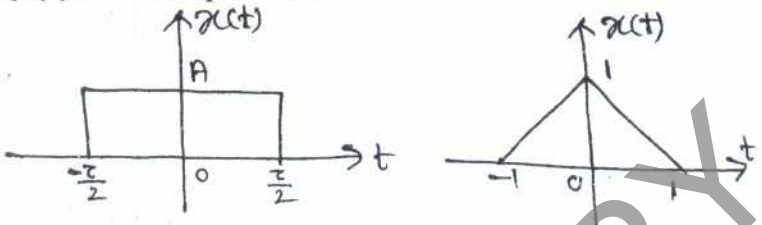

Fig.Q5(c)

6 a. A particular discrete-time system has input x(n) and output y(n). The Fourier transforms of these signals are related by the equation $Y(e^{jw}) = ZX(e^{jw}) + e^{-jw}X(e^{jw}) - \frac{dX(e^{jw})}{dw}$.

Is the system linear? Clearly justify your answer. What is y(n) if $x(n) = \delta(n)$? Is the system causal? **(06 Marks)**

b. Consider a causal and state LTI system S having frequency response $H(w) = \frac{jw+4}{6-w^2+5jw}$.

i) Obtain the differential equation for the system.
ii) Determine the impulse response h(t) of S
iii) What is the output of S when the input is $x(t) = e^{-4t}u(t) - te^{-4t}u(t)$ **(10 Marks)**

c. If $x(t) \leftrightarrow X(f)$
Show that $x(t)\cos w_0 t \leftrightarrow \frac{1}{2}[X(f-f_0) + X(f+f_0)]$ where $w_0 = 2\pi f_0$. **(04 Marks)**

7 a. What is region of convergence of X(z), where X(z) is the z-transform of x(n). State all the properties of R.O.C. **(05 Marks)**

b. Determine the Z-transform of the following sequences including R.O.C.

i) $\delta(n+5)$ ii) $\left(\frac{1}{2}\right)^{n+1} u(n+3)$ iii) $\left(-\frac{1}{3}\right)^n u(-n-2)$

iv) $2^n u(-n) + \left(\frac{1}{4}\right)^n u(n-1)$ v) $\alpha^{|n|}$ for $o < \alpha < 1$. **(15 Marks)**

8 a. State and prove time reversal property. Find value theorem of Z-transform. Using suitable properties, find the Z-transform of the sequences

i) $(n-2)\left(\frac{1}{3}\right)^{n-2} u(n-2)$ ii) $(n+1)\left(\frac{1}{2}\right)^{n+1} \cos w_0(n+1)u(n+1)$ **(10 Marks)**

b. Consider a system whose difference equation is $y(n-1) + 2y(n) = x(n)$
i) Determine the zero-input response of this system, if $y(-1) = 2$.
ii) Determine the zero state response of the system to the input $x(n)=(1/4)^n u(n)$.
iii) What is the frequency response of this system?
iv) Find the unit impulse response of this system. **(10 Marks)**

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06EC45

Fourth Semester B.E. Degree Examination, May/June 2010
Fundamentals of HDL

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions, selecting at least TWO questions from each part.***

PART – A

1 a. Explain the behavioral and structural description types of HDL programming, with examples and keywords used. **(10 Marks)**

b. Explain the following data types:
i) Physical std_logic and bit_vector in VHDL
ii) Nets, parameters and registers in verilog. **(10 Marks)**

2 a. How do you assign a delay time to the signal assignment statement? Explain the dataflow model of 2×1 multiplexer in VHDL and verilog. **(10 Marks)**

b. Explain the use of data type vectors with dataflow description of 2×2 unsigned combinational array multiplier in VHDL and verilog. **(10 Marks)**

3 a. Differentiate between signal and variable assignment statement in VHDL. Write VHDL programs for behavioral description of D-latch using signal assignment and variable assignment. **(10 Marks)**

b. Explain the formats of for-loop and while-loop statements in VHDL and verilog. **(06 Marks)**

c. Write verilog description for a 4-bit priority encoder. **(04 Marks)**

4 a. Explain the binding in the following, with example:
i) Between entity and component in VHDL
ii) Between two modules in verilog. **(10 Marks)**

b. Write the HDL programs for N+1 bit magnitude comparator using
i) generate and generic in VHDL ii) generate and parameter in verilog. **(10 Marks)**

PART – B

5 a. Explain the use of procedure (in VHDL) and task (in verilog) with description of full adder, using half adders. **(10 Marks)**

b. Explain the file declaration and built in procedures for file handling in VHDL. **(10 Marks)**

6 a. How to attach a package to the VHDL module? Explain with an example. **(08 Marks)**

b. What is the need of mixed type descriptions? Write description of 16×8 SRAM in VHDL and verilog. **(12 Marks)**

7 a. How to imvoke a VHDL entity from a verilog module. Explain with an example. **(10 Marks)**

b. Write a HDL program for mixed language descriptions of a JK-flip-flop with a clear input. **(10 Marks)**

8 a. Write a flow diagram of synthesis. Explain its steps. **(08 Marks)**

b. Write VHDL code for signal assignment statement y = 2 * x + 3. Show the synthesized logic symbol and gate level diagram. Write structural code in verilog using the gate level diagram. **(12 Marks)**

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06EC46

Fourth Semester B.E. Degree Examination, May/June 2010
Linear IC's and Applications

Time: 3 hrs. Max. Marks:100

Note: *1. Answer any FIVE full questions, selecting at least TWO questions from each part.*
2. Use of resistor, capacitor standard values list and op-amp data sheets is permitted.

PART – A

1 a. Define the following parameters :
i) Input offset voltage
ii) CMRR
iii) Slewrate.
Mention their typical values for op-amp 741. **(06 Marks)**

b. Sketch the circuit of a two-input inverting summing amplifier. Explain the operation of the circuit and derive the equation for the output voltage. **(08 Marks)**

c. A non-inverting amplifier is to amplify a 100 mV signal to a level of 3V. Using a 741 op-amp, design a suitable circuit. **(06 Marks)**

2 a. Sketch the circuit of a high Z_{in} capacitor coupled voltage follower. Briefly explain its operation and show that the input impedance is very high compared to the capacitor coupled voltage follower. **(06 Marks)**

b. Explain how the upper cutoff frequency can be set for inverting and on-inverting amplifiers. **(06 Marks)**

c. Design Z_{in} capacitor coupled voltage follower using an op-amp having lower cutoff frequency of 50 Hz and maximum input bias current of 500 nA. The load resistance is 3.6 KΩ. If the open loop gain is 2×10^5, find ideal value of input impedance of the circuit. **(08 Marks)**

3 a. With a neat sketch, explain the working of a lag compensation network. Show how it affects the frequency response of an op-amp. **(08 Marks)**

b. List the precautions to be observed for op-amp circuit stability. **(08 Marks)**

c. Determine the upper cutoff frequency and the maximum distortion free output amplitude for a voltage follower when a 741 op-amp is used. **(04 Marks)**

4 a. With a neat sketch, explain the working of a precision voltage source using op-amp with a zener diode. Derive an expression relating V_o and V_z. **(08 Marks)**

b. Draw the circuit of an instrumentation amplifier. Explain its characteristics. Also show how the voltage gain can be varied. **(08 Marks)**

c Determine the range of resistance R_G for a LH0036 IC instrumentation amplifier to give a voltage gain adjustable from 30 to 300. **(04 Marks)**

PART – B

5 a. Draw and explain an op-amp sample and hold circuit with signal, control and output waveforms. **(08 Marks)**

b. With a neat sketch, explain the working of Wein-Bridge Oscillator circuit. **(06 Marks)**

c. Using a 741 op-amp with a supply of ± 12V, design a phase-shift oscillator to have an output frequency of 3.5 KHz. **(06 Marks)**

6 a. What are the advantages of active filters over passive filters? **(04 Marks)**

b. Sketch the circuit of a second order active high pass filter, explain its working. **(08 Marks)**

c. An inverting Schmitt trigger circuit is to have UTP = 0 and LTP = 2.5 V. Design a suitable circuit using a bipolar op-amp and a ± 18 V supply. **(08 Marks)**

7 a. Explain the application of IC 723 as basic low voltage regulator. **(06 Marks)**

b. Explain the principle of switch mode power supplies. Enumerate their advantages and disadvantages. **(08 Marks)**

c. Using 7805 design a current source to deliver 0.2 A current to a 22 Ω, 10W loads. Take quiescent current as 4.2 mA. **(06 Marks)**

8 a. Draw and explain the functional diagram of 555 timer. **(06 Marks)**

b. With a sketch, explain the working of R-2R ladder DAC. **(08 Marks)**

c. An 8 bit ADC outputs all 1's when $V_i = 2.55$ V. Find its :

i) Resolution in mV/LSB and

ii) Digital output when $V_i = 1.28$ V. **(06 Marks)**

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MATDIP401

Fourth Semester B.E. Degree Examination, May/June 2010
Advanced Mathematics – II

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

1 a. Find the projection of the line AB on CD where A = (1, 2, 3), B = (-1, 0, 2), C = (1, 4, 2), D = (2, 0, -1). **(06 Marks)**

b. Find the angle between two lines whose direction cosines are given by $l + 3m + 5n = 0$ and $2mn - 6nl - 5lm = 0$. **(07 Marks)**

c. A line makes angles $\alpha, \beta, \gamma, \delta$ with diagonals of a cube. Prove that $\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta = \frac{4}{3}$. **(07 Marks)**

2 a. Find the equation of the plane passing through the points (3, 1, 2) and (3, 4, 4) and perpendicular to $5x + y + 4z = 0$. **(06 Marks)**

b. Show that the points (2, 2, 0), (4, 5, 1), (3, 9, 4) and (0, -1, -1) are coplanar. Find the equation of the plane containing them. **(07 Marks)**

c. Find the equation of a straight line through (7, 2, -3) and perpendicular to each of the lines. $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$ and $\frac{x+2}{4} = \frac{y-3}{5} = \frac{z-4}{6}$. **(07 Marks)**

3 a. Show that the position vectors of the vertices of a triangle $\vec{a} = 3\left(\sqrt{3}\,\hat{i} - \hat{j}\right)$, $\vec{b} = 6\hat{j}$ $\vec{c} = 3\left(\sqrt{3}\,\hat{i} + \hat{j}\right)$ form an isosceles triangle. **(06 Marks)**

b. A particle moves along the curve $\vec{r} = 3t^2\hat{i} + (t^3 - 4t)\hat{j} + (3t + 4)\hat{k}$. Find the components of velocity and acceleration at t = 2 in the direction $\hat{i} - 2\hat{j} + 2\hat{k}$. **(07 Marks)**

c. Find the angle between the normals to the surfaces $x^2y^2 = z^4$ at (1, 1, 1) and (3, 3, -3). **(07 Marks)**

4 a. Find the directional derivatives of the function $\phi = xyz$ along the direction of the normal to the surface $xy^2 + yz^2 + zx^2 = 3$ at the point (1, 1, 1). **(06 Marks)**

b. Find the div $\vec{F}$ and curl $\vec{F}$ where $\vec{F} = \nabla(x^3 + y^3 + z^3 - 3xyz)$. **(07 Marks)**

c. If $\vec{v} = 2xy\,\hat{i} + 3x^2y\,\hat{j} - 3ayz\,\hat{k}$ is solenoidal at (1, 1, 1), find a. **(07 Marks)**

5 a. Find the unit normal vector to the surface $xy + x + zx = 3$ at (1, 1, 1). **(06 Marks)**

b. Find the constants 'a', 'b', 'c' such that the vector field
$(\text{Sin } y + az)\ \hat{i} + (bx \text{ Cos } y + z)\ \hat{j} + (x + cy)\ \hat{k}$
is irrotational. Also find the scalar field ϕ such that $\vec{F} = \nabla\phi$. **(07 Marks)**

c. Prove that $\nabla^2 (\log r) = \frac{1}{r^2}$ where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $r = |\vec{r}|$. **(07 Marks)**

6 a. Find the Laplace transform of Sin 2t Sin 3t. **(05 Marks)**

b. Find $L\left[\frac{(1-e^t)}{t}\right]$. **(05 Marks)**

c. Find $L\left[e^{-t}(3 \text{ Sinh } 2t - 2 \text{ Cosh } 3t)\right]$. **(05 Marks)**

d. Find the Laplace transform of $f(t) = \begin{cases} t/\lambda & \text{when } 0 < t < \lambda \\ 1 & \text{when } t > \lambda \end{cases}$. **(05 Marks)**

7 a. Evaluate $\int_0^\infty \frac{\text{Sin}t}{t} dt$ using Laplace transform. **(05 Marks)**

b. Find the inverse Laplace transform of $\frac{1}{(s^2 + 3s + 2)(s + 3)}$. **(05 Marks)**

c. Find $L^{-1}\left[\frac{s-1}{s^2 - 6s + 25}\right]$. **(05 Marks)**

d. Find $L^{-1}\left[\log\left\{\frac{s^2+1}{s^2 - s}\right\}\right]$. **(05 Marks)**

8 a. Find $L^{-1}\left[\frac{1}{s^2(s+5)}\right]$ using convolution theorem. **(10 Marks)**

b. Solve the differential equation $y'' + 2y' + y = 6te^{-t}$ under the condition $y(0) = 0 = y'(0)$ using Laplace transform. **(10 Marks)**

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